

*ELECTROMAGNETIC  
STRUCTURE OF CHARMED  
BARYONS IN LATTICE QCD*

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*Talk based on JHEP05(2014) 125*

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# *OUTLINE*

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- *Motivation*
- *Electromagnetic (EM) form factors*
  - *Parameterisation*
  - *Lattice Formulation*
- *Simulation Details*
- *Results*
- *Conclusions and outlook*

# *MOTIVATION*

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- *Probe the hadron structure*
- *size, charge radii, magnetic moment*
- *Effect of heavy quarks*
- *Previous work: heavy quark shrinks the mesons*

*kuc, G. Erkol, M. Oka, A. Ozpineci, T.T. Takahashi PLB 719*

$$\langle r^2 \rangle_D = 0.138 \text{ fm}^2$$

$$\langle r^2 \rangle_{D^*} = 0.185 \text{ fm}^2$$

$$\langle r^2 \rangle_\pi = 0.452 \text{ fm}^2$$

# EM FORM FACTORS

$$\langle \mathcal{B}(p) | V_\mu | \mathcal{B}(p') \rangle = \bar{u}(p) \left[ \gamma_\mu F_{1,\mathcal{B}}(q^2) + i \frac{\sigma_{\mu\nu} q^\nu}{2m_{\mathcal{B}}} F_{2,\mathcal{B}}(q^2) \right] u(p)$$

*Sachs FFs*

$$G_{E,\mathcal{B}}(q^2) = F_{1,\mathcal{B}}(q^2) + \frac{q^2}{4m_{\mathcal{B}}^2} F_{2,\mathcal{B}}(q^2)$$
$$G_{M,\mathcal{B}}(q^2) = F_{1,\mathcal{B}}(q^2) + F_{2,\mathcal{B}}(q^2)$$

## Charge Radii & Magnetic Moment

$$\langle r_{E,M}^2 \rangle = -\frac{6}{G_{E,M}(0)} \frac{d}{dQ^2} G_{E,M}(Q^2) \Big|_{Q^2=0}$$
$$G_{E,M}(Q^2) = \frac{G_{E,M}(0)}{(1+Q^2/\Lambda_{E,M}^2)^2}$$
$$\Rightarrow \langle r_{E,M}^2 \rangle = \frac{12}{\Lambda_{E,M}^2}$$
$$\mu_B = G_M(0) \left( \frac{m_N}{2m_B} \right) \mu_N$$

# EM FORM FACTORS

## Lattice Formulation

$$\langle C^{\mathcal{B}}(t; \mathbf{p}; \Gamma_4) \rangle = \sum_{\mathbf{x}} e^{-i\mathbf{p} \cdot \mathbf{x}} \Gamma_4^{\alpha\alpha'} \langle \text{vac} | T[\eta_{\mathcal{B}}^\alpha(x) \bar{\eta}_{\mathcal{B}}^{\alpha'}(0)] | \text{vac} \rangle$$

$$\langle C^{\mathcal{BV}_\mu \mathcal{B}'}(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma) \rangle = -i \sum_{\mathbf{x}_2, \mathbf{x}_1} e^{-i\mathbf{p} \cdot \mathbf{x}_2} e^{i\mathbf{q} \cdot \mathbf{x}_1} \Gamma^{\alpha\alpha'} \langle \text{vac} | T[\eta_{\mathcal{B}}^\alpha(x_2) V_\mu(x_1) \bar{\eta}_{\mathcal{B}'}^{\alpha'}(0)] | \text{vac} \rangle$$

$$R(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma; \mu) = \frac{\langle C^{\mathcal{BV}_\mu \mathcal{B}'}(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma) \rangle}{\langle C^{\mathcal{BB}}(t_2; \mathbf{p}'; \Gamma_4) \rangle} \\ \times \left[ \frac{\langle C^{\mathcal{BB}}(t_2 - t_1; \mathbf{p}; \Gamma_4) \rangle \langle C^{\mathcal{BB}}(t_1; \mathbf{p}'; \Gamma_4) \rangle \langle C^{\mathcal{BB}}(t_2; \mathbf{p}'; \Gamma_4) \rangle}{\langle C^{\mathcal{BB}}(t_2 - t_1; \mathbf{p}'; \Gamma_4) \rangle \langle C^{\mathcal{BB}}(t_1; \mathbf{p}; \Gamma_4) \rangle \langle C^{\mathcal{BB}}(t_2; \mathbf{p}; \Gamma_4) \rangle} \right]^{1/2}$$

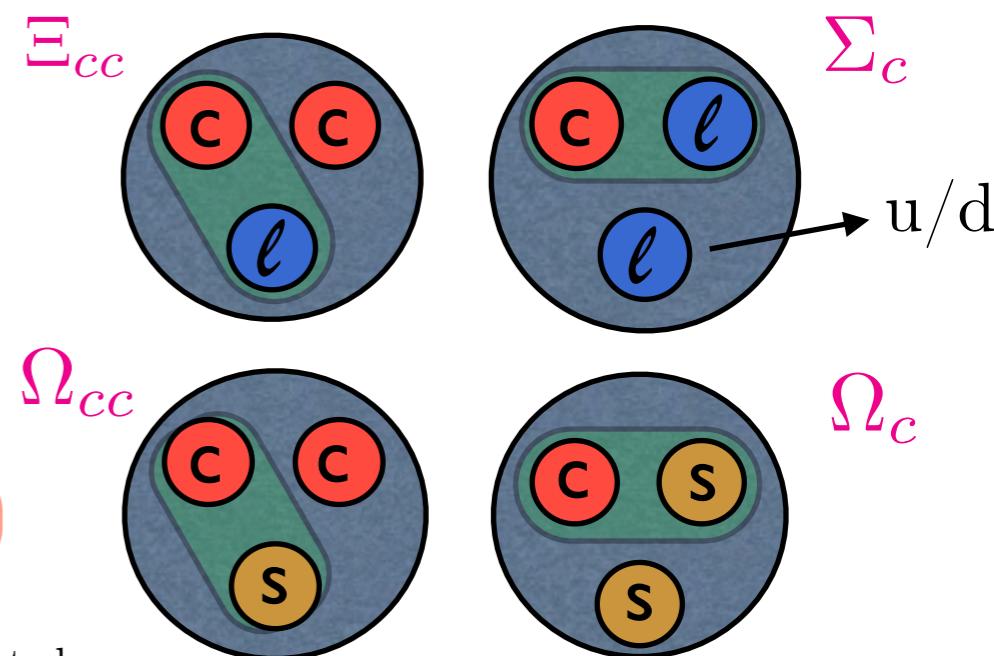
$$R(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma; \mu) \xrightarrow[t_2 - t_1 \gg a]{t_1 \gg a} \Pi(\mathbf{p}', \mathbf{p}; \Gamma; \mu)$$

$$\Pi(\mathbf{0}, -\mathbf{q}; \Gamma_4; \mu = 4) = \left[ \frac{(E_{\mathcal{B}} + m_{\mathcal{B}})}{2E_{\mathcal{B}}} \right]^{1/2} G_{E, \mathcal{B}}(q^2)$$

$$\Pi(\mathbf{0}, -\mathbf{q}; \Gamma_j; \mu = i) = \left[ \frac{1}{2E_{\mathcal{B}}(E_{\mathcal{B}} + m_{\mathcal{B}})} \right]^{1/2} \epsilon_{ijk} q_k G_{M, \mathcal{B}}(q^2)$$

G(0) should be extrapolated  
from higher momenta

$$\eta_{\Xi_{cc}}(x) = \epsilon^{ijk} [c^{Ti}(x) C \gamma_5 \ell^j(x)] c^k(x) \\ \eta_{\Sigma_c}(x) = \epsilon^{ijk} [\ell^{Ti}(x) C \gamma_5 c^j(x)] \ell^k(x) \\ \eta_{\Omega_{cc}}(x) = \epsilon^{ijk} [c^{Ti}(x) C \gamma_5 s^j(x)] c^k(x) \\ \eta_{\Omega_c}(x) = \epsilon^{ijk} [s^{Ti}(x) C \gamma_5 c^j(x)] s^k(x)$$



# *SIMULATION DETAILS*

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1. PACS-CS generated  $32^3 \times 64$ ,  $\beta=1.9$ , 2+1 flavor (u/d,s) lattices [Phys. Rev. D79 \(034503\)](#)

II.  $\kappa_c$  tuned to 1S  $M_{\eta-J/\psi}$ ,  $M_{D-D^*}$ ,  $M_{D_s-D_s^*}$

I. Gauge action: *Iwasaki*, Fermion action:  
*Clover*

3. Point-split (conserved) vector current:  
renormalisation not necessary

II.  $a = 0.0907(13)$  fm,  $a^{-1} = 2.176(31)$  GeV

4. Multiple Shell source - Wall sink pairs

III. Box Size:  $(2.9 \text{ fm})^3 \times 5.8 \text{ fm}$

I.  $t = 12 a$  separation (more on next slide)

IV.  $\kappa_{ud} = 0.13700, 0.13727, 0.13754, 0.13770,$

II. Smearing:  $\langle r_l \rangle \sim 0.5 \text{ fm}$ ,  $\langle r_c \rangle \sim \langle r_l \rangle / 3$

i.  $m_\pi \sim 700, 570, 410, 300 \text{ MeV}$

III. Wall sinks: no need for sequential  
inversions, *caveat: increased noise!*

V.  $\kappa_s = 0.13640$ ,  $\kappa_c = 0.1246$

i. Coulomb gauge fix: wall smearing is  
gauge dependent

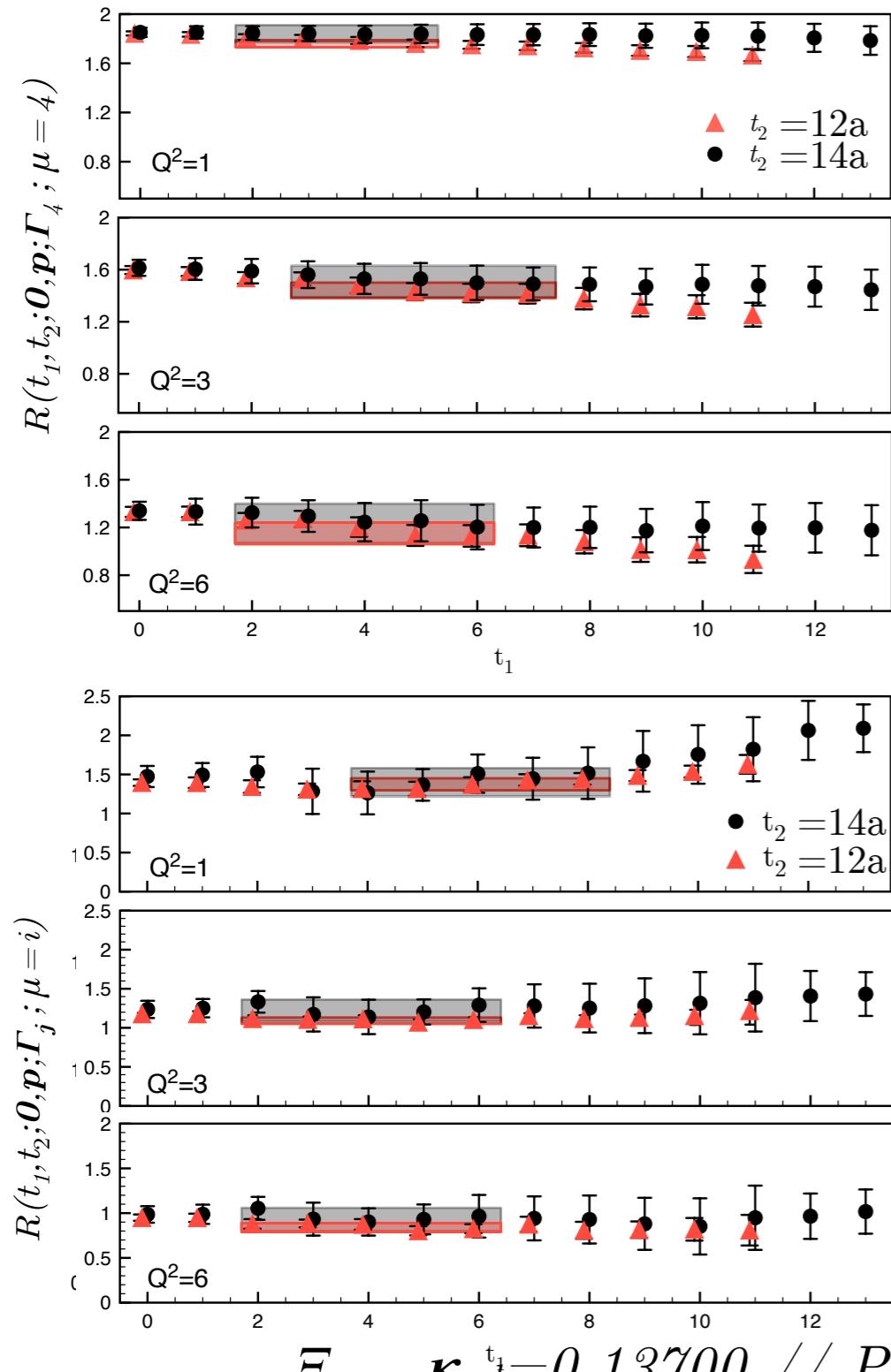
2. Clover action for all valance quarks

I.  $c_E = c_B = 1/(u_0)^3$

5. Single-elimination Jackknife analysis

# SIMULATION DETAILS

## Smallest time separation possible



$\Xi_{cc} - \kappa_{ud}^{t_1} = 0.13700$  // Plateau fits = 100 confs // SOI = 30 conf

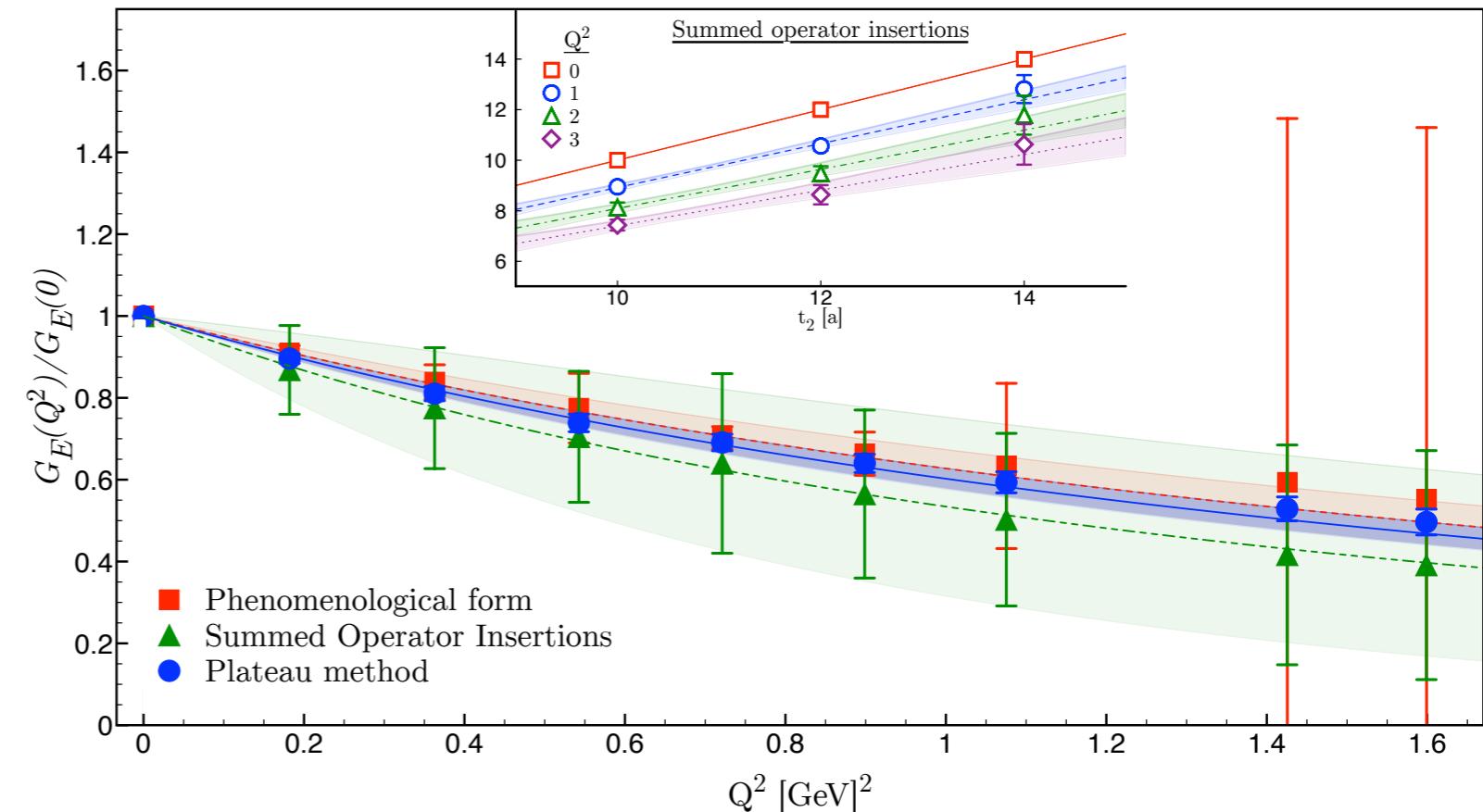
## Excited State Contamination

### ■ Phenomenological Form

$$R(t_2, t_1) = G_{E,M} + b_1 e^{-\Delta t_1} + b_2 e^{-\Delta(t_2 - t_1)}$$

### ■ Summed Operator Insertions

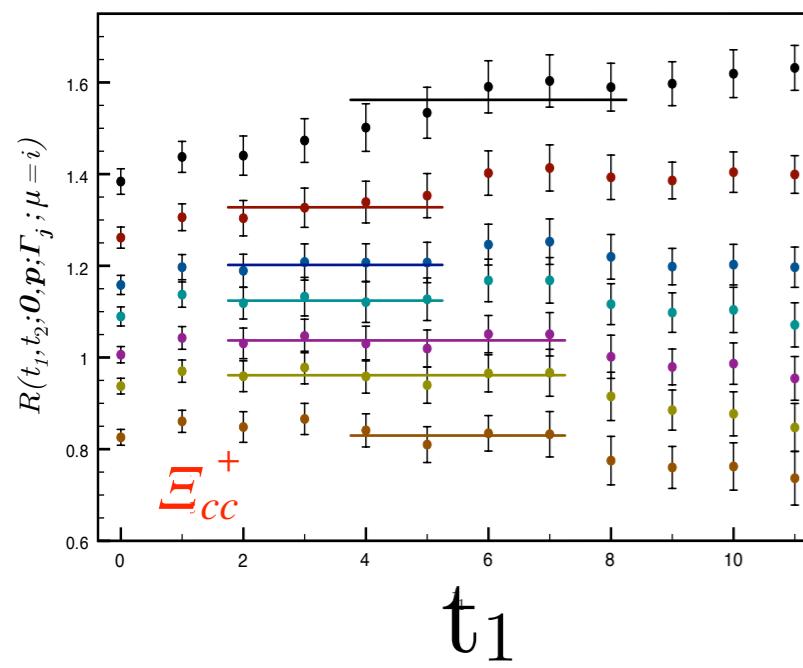
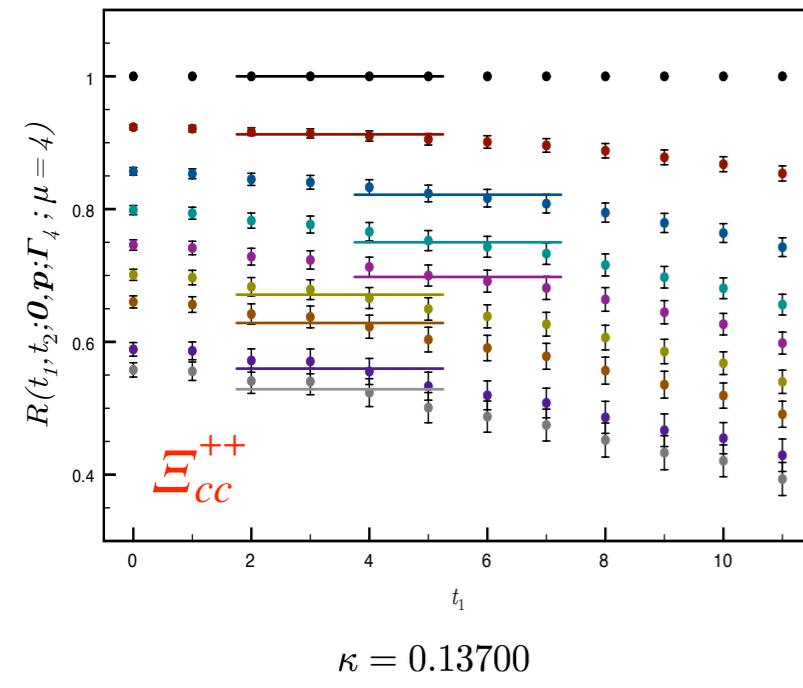
$$S(t_s) = \sum_{t=0}^{t_s} R(\vec{q}, t, t_s) \rightarrow c(\Delta, \Delta') + t_s \left( G_{E,M} + \mathcal{O}(e^{-\Delta t_s}) + \mathcal{O}(e^{-\Delta' t_s}) \right)$$



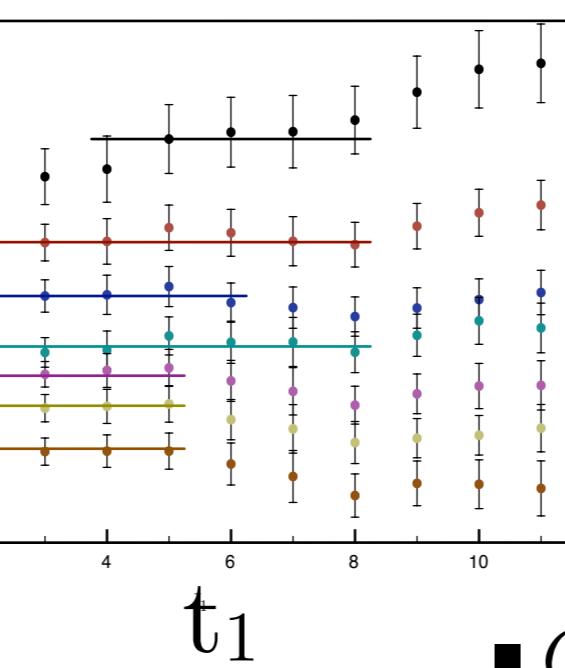
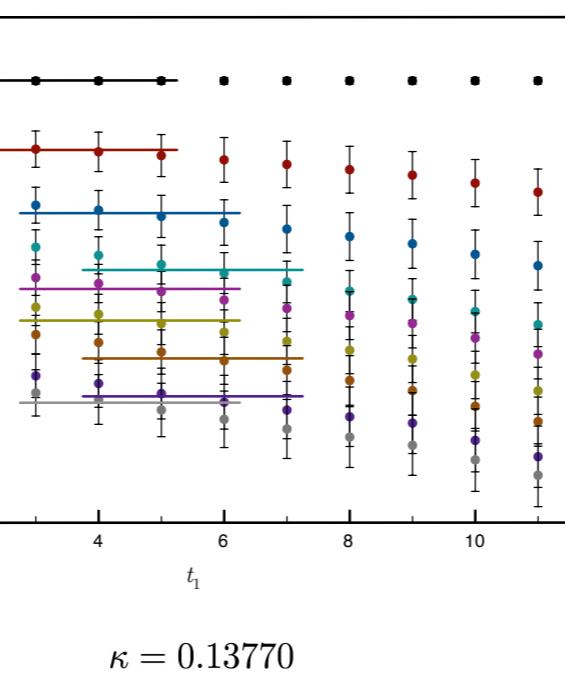
$\Xi_{cc} - \kappa_{ud}^{t_1} = 0.13700$  // Plateau fits = 100 confs // SOI = 30 conf

- $\kappa_{ud} = 0.13700/70$
- p-value criteria

$m_\pi = \textcolor{red}{700}$  MeV

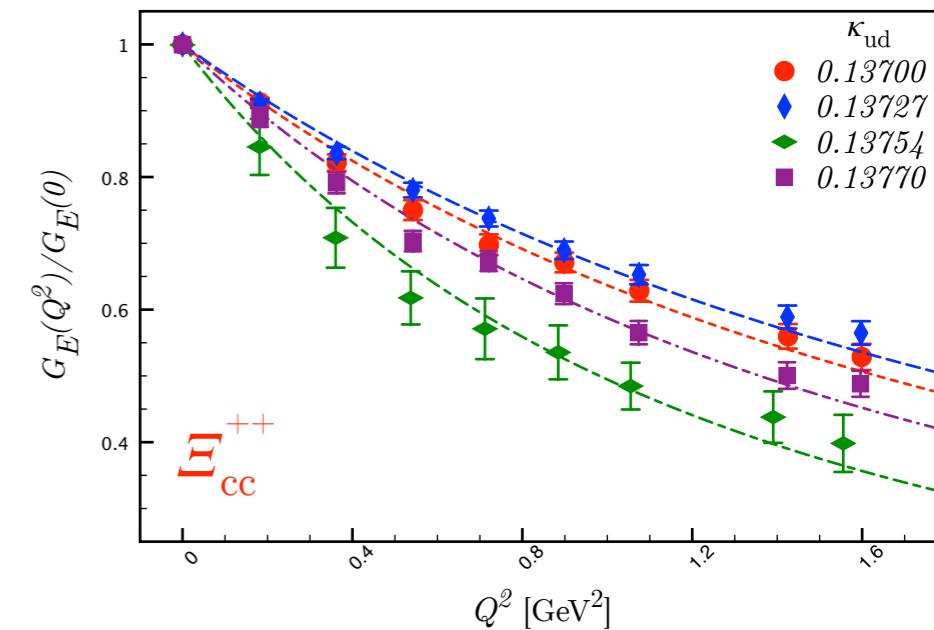


300 MeV

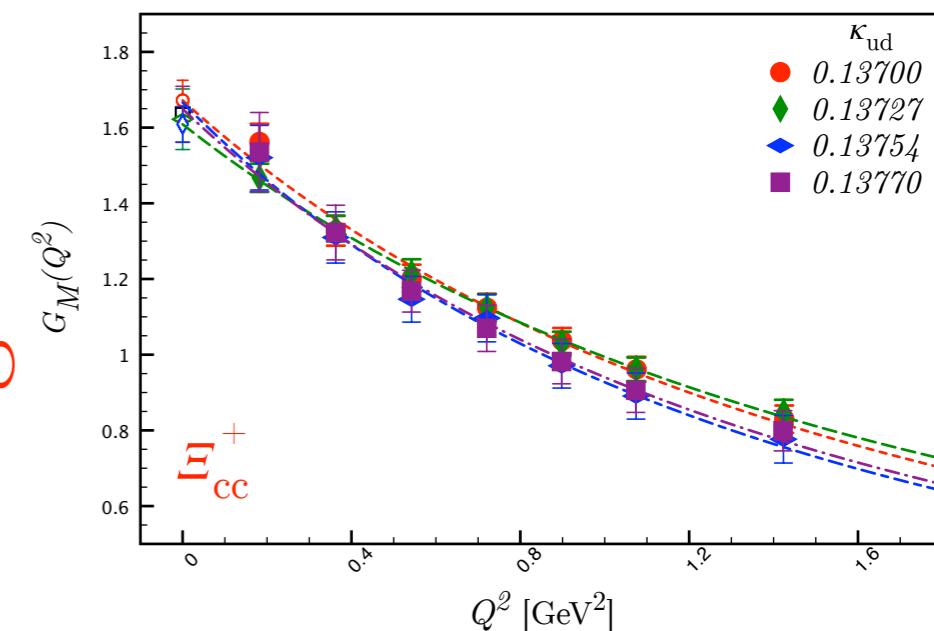


# RESULTS

plateaus and form factor fits:  $\Xi_{cc}$



## Electric



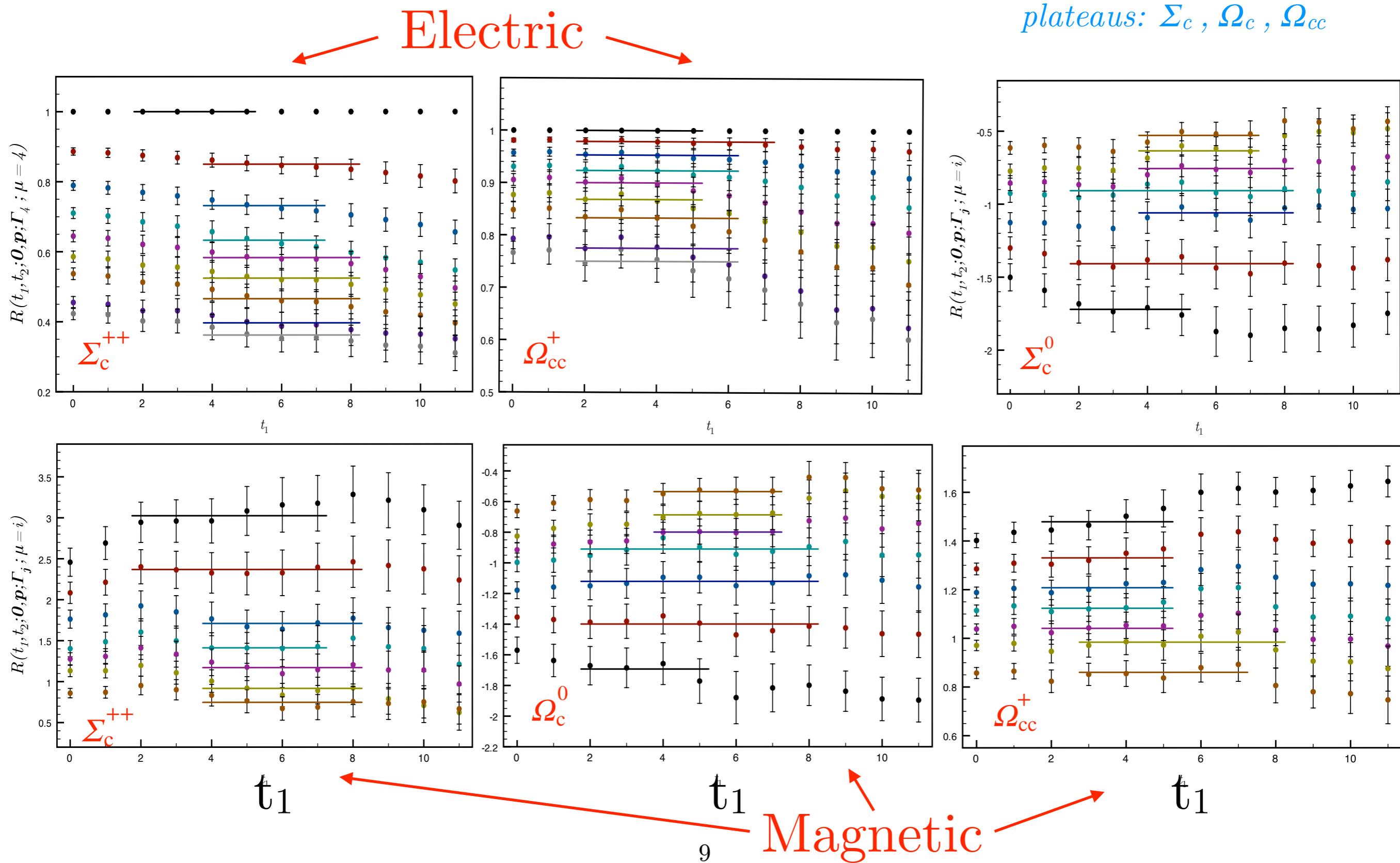
## Magnetic

- Good fits to the dipole form
- EFFs are normalised to unit charge

- $\kappa_{ud} = 0.13700$
- 9 (7) 4-mom insertions for electric (magnetic) form factor
- p-value criteria

# RESULTS

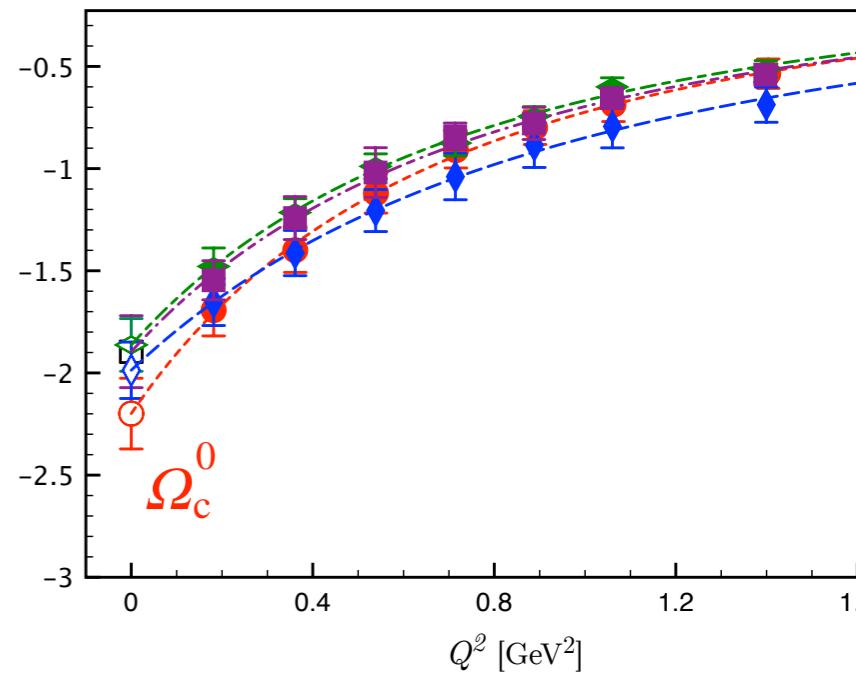
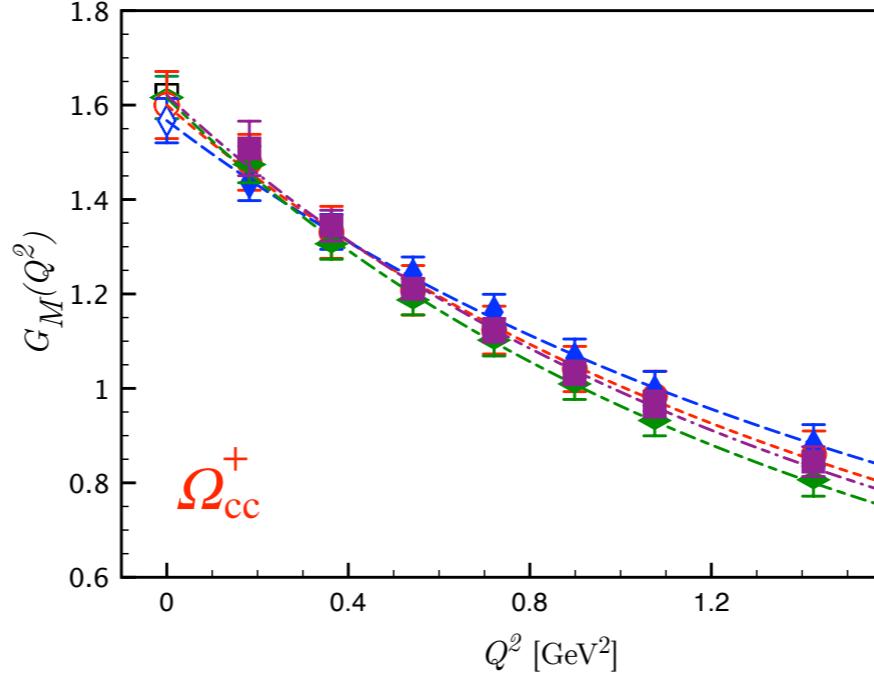
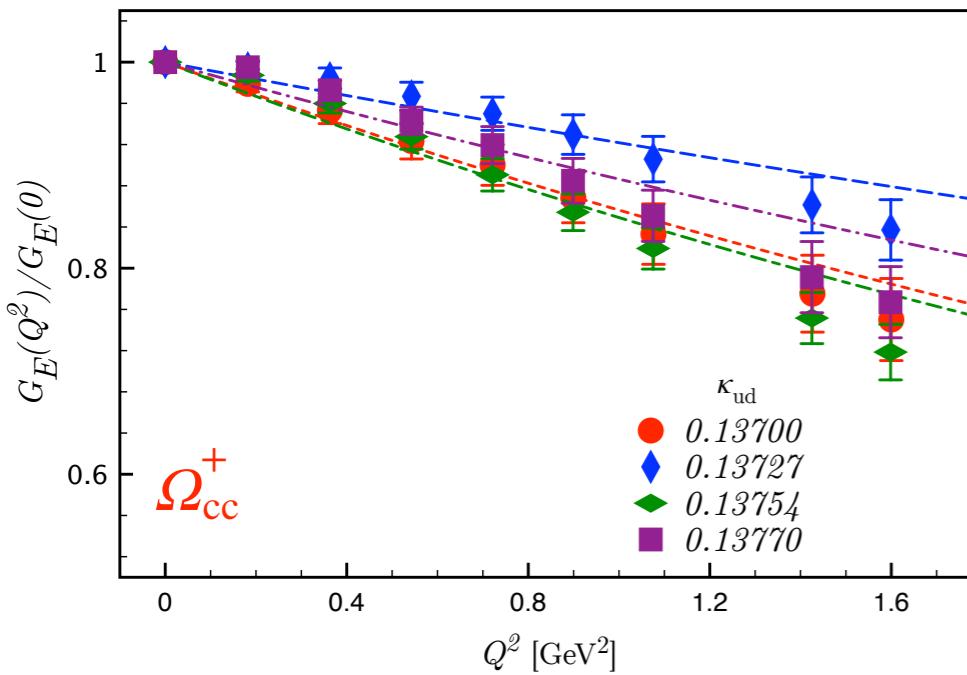
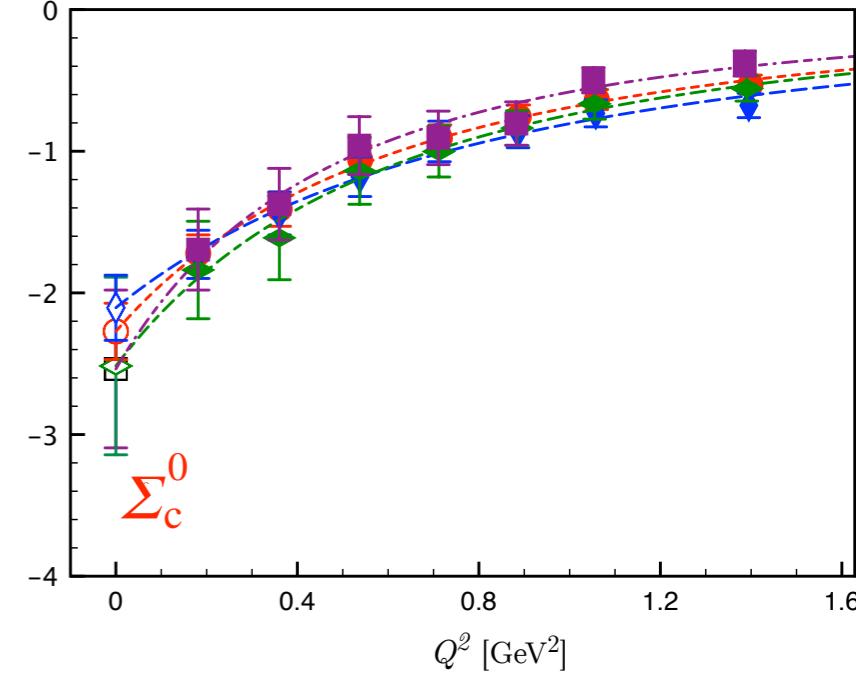
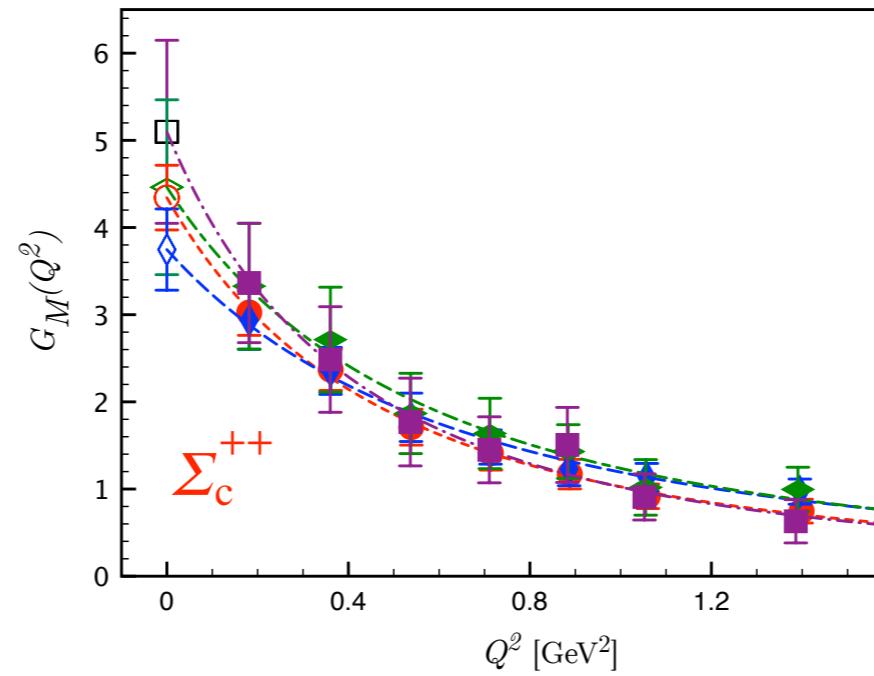
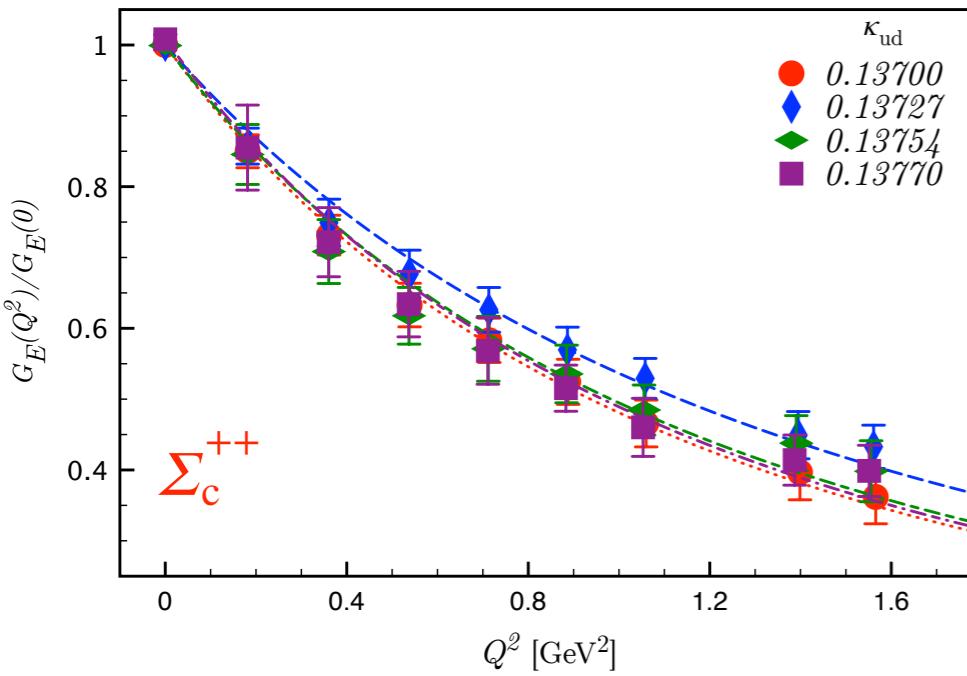
plateaus:  $\Sigma_c$ ,  $\Omega_c$ ,  $\Omega_{cc}$



- $\kappa_{ud} = 0.13700$
- Dipole Form
- 9 (7) 4-mom insertions for electric (magnetic) form factor

# RESULTS

form factors:  $\Sigma_c$ ,  $\Omega_c$ ,  $\Omega_{cc}$



Electric

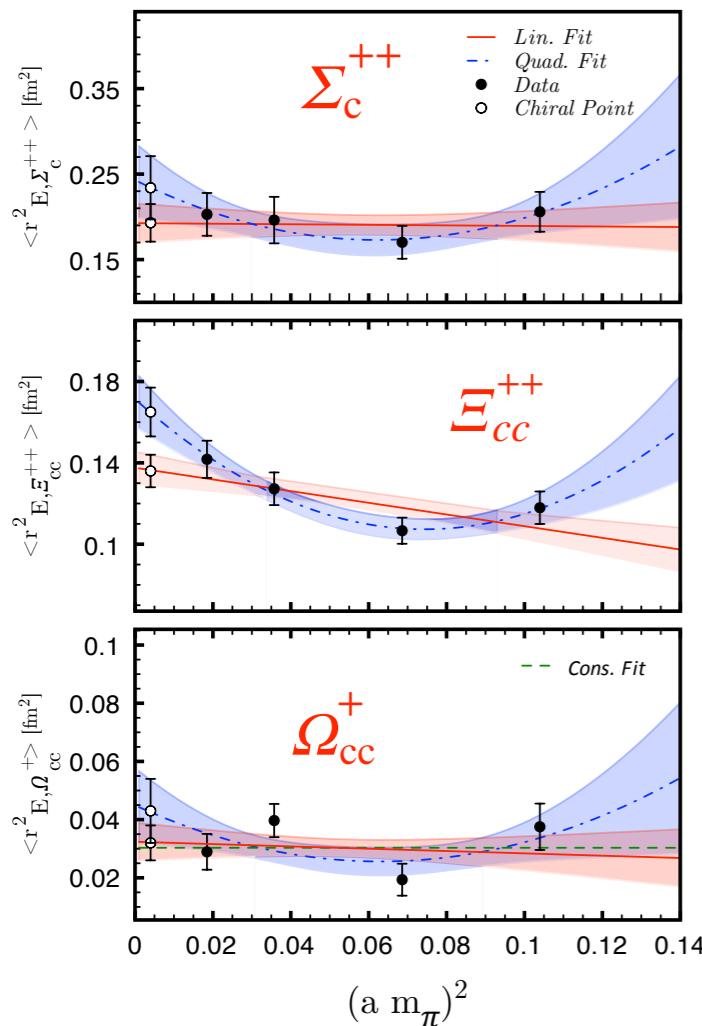
10

Magnetic

$\kappa_{ud}(137-)$	70	54	27	00	
stat.	( $\Sigma_c$ , $\Xi_{cc}$ )	170	150	100	100
	( $\Omega_c$ , $\Omega_{cc}$ )	130	100	100	100

# CHIRAL FITS

$\langle r^2_E \rangle$

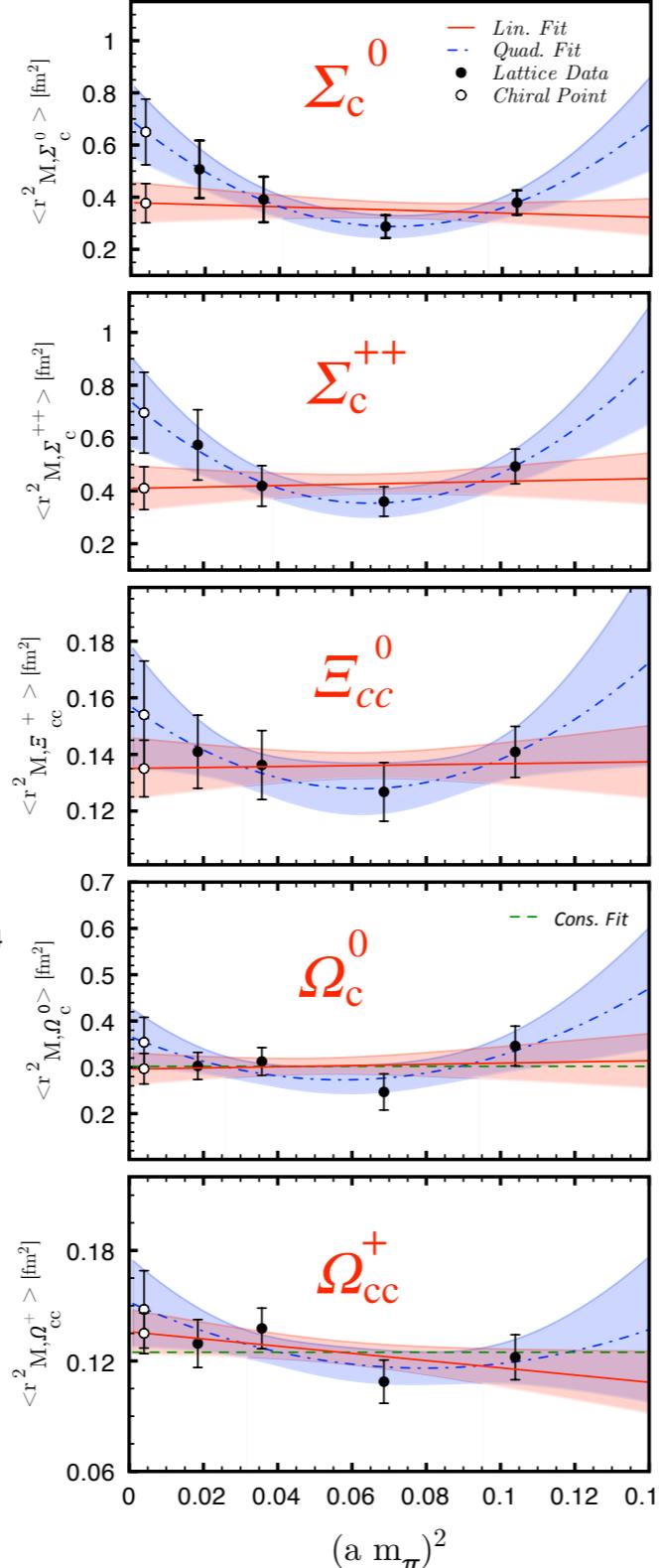


$m_\pi \sim 300 \text{ } 410 \text{ } 570 \text{ } 700 \text{ MeV}$

	Fit Form	Quad. Fit
uuc	$\langle r^2_{E,\Sigma_c^{++}} \rangle$	0.234(37)
ucc	$\langle r^2_{E,\Xi_{cc}^{++}} \rangle$	0.165(12)
SCC	$\langle r^2_{E,\Omega_{cc}^{+}} \rangle$	0.043(11)
dcc	$\langle r^2_{E,\Xi_{cc}^{+}} \rangle = 0.042(9)$	

\* $\langle r^2_{E,p} \rangle = 0.770 \text{ fm}^2$

$\langle r^2_M \rangle$



Fit Form | Quad. Fit

uuc $\langle r^2_{M,\Sigma_c^{++}} \rangle$	0.696(153)
ddc $\langle r^2_{M,\Sigma_c^0} \rangle$	0.650(126)
dcc $\langle r^2_{M,\Xi_{cc}^{+}} \rangle$	0.154(19)
SCC $\langle r^2_{M,\Omega_{cc}^{+}} \rangle$	0.148(21)
SSC $\langle r^2_{M,\Omega_c^0} \rangle$	0.354(54)

\* $\langle r^2_{M,p} \rangle = 0.604 \text{ fm}^2$

$\langle r^2_{M,n} \rangle = 0.862 \text{ fm}^2$

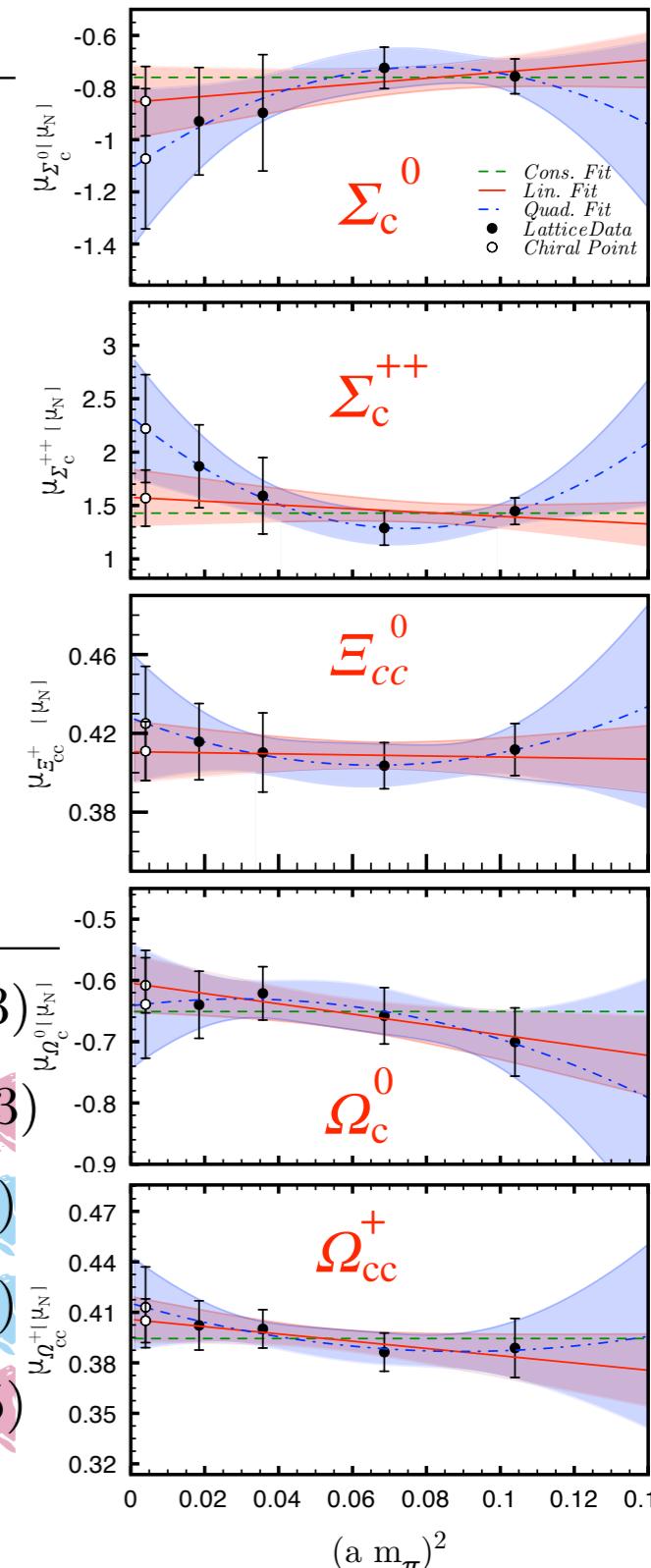
Fit Form | Lin. Fit

uuc $\mu_{\Sigma_c^{++}}$	1.569(253)
ddc $\mu_{\Sigma_c^0}$	-0.852(133)
dcc $\mu_{\Xi_{cc}^{+}}$	0.411(15)
SCC $\mu_{\Omega_{cc}^{+}}$	0.405(13)
SSC $\mu_{\Omega_c^0}$	-0.608(45)

\* $\mu_p = 2.793 \mu_N$

$\mu_n = -1.913 \mu_N$

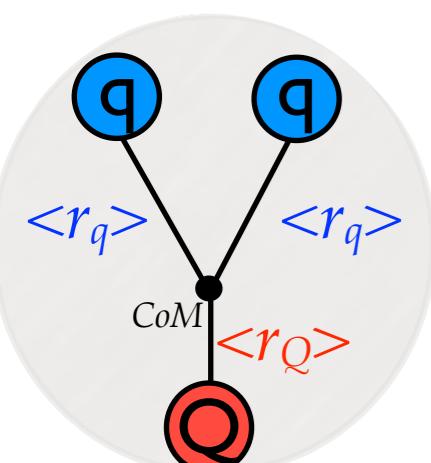
$\mu$



\* PDG values

# QUARK CONTRIBUTIONS

Baryon	Fit Form	$\langle r_E^2 \rangle_q$	$\langle r_E^2 \rangle_Q$	$\langle r_M^2 \rangle_q$	$\langle r_M^2 \rangle_Q$	$\mu_q$	$\mu_Q$
$\Sigma_c^{0,++}$	Lin. Fit	[fm <sup>2</sup> ]	[fm <sup>2</sup> ]	[fm <sup>2</sup> ]	[fm <sup>2</sup> ]	$[\mu_N]$	$[\mu_N]$
		0.347(49)	0.032(18)	0.403(67)	0.098(80)		
$\Xi_{cc}^{+,++}$	Quad. Fit	0.390(86)	0.066(32)	0.604(118)	0.236(183)	$2.369(362)$	$-0.099(21)$
$\Omega_c^0$	Lin. Fit	0.386(33)	0.068(5)	0.426(60)	0.082(6)	$-0.410(51)$	$0.430(8)$
		0.410(46)	0.095(9)	0.612(115)	0.089(11)		
$\Omega_{cc}^+$	Lin. Fit	0.330(32)	0.064(10)	0.398(44)	0.056(19)	$1.710(150)$	$-0.099(14)$
		0.398(52)	0.069(22)	0.484(70)	0.054(38)		
$\Omega_{cc}^+$	Quad. Fit	0.287(31)	0.078(7)	0.350(44)	0.095(9)	$-0.370(26)$	$0.441(12)$
		0.422(51)	0.104(13)	0.534(72)	0.101(16)		

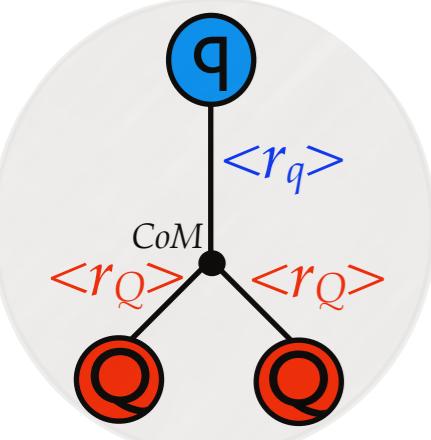


$$\langle r^2 \rangle_{E,\Sigma_c^{0+}} = 0.234(37) \text{ fm}^2$$

$$\langle r^2 \rangle_{E,\Xi_{cc}^{++}} = 0.042(9) \text{ fm}^2$$

$$\langle r^2 \rangle_{E,\Xi_{cc}^{0+}} = 0.165(12) \text{ fm}^2$$

$$\langle r^2 \rangle_{E,\Omega_{cc}^+} = 0.043(11) \text{ fm}^2$$



$$\langle r^2 \rangle_{M,\Sigma_c^0} = 0.650(126) \text{ fm}^2$$

$$\langle r^2 \rangle_{M,\Sigma_c^{0+}} = 0.696(153) \text{ fm}^2$$

$$\langle r^2 \rangle_{M,\Xi_{cc}^+} = 0.154(19) \text{ fm}^2$$

$$\langle r^2 \rangle_{M,\Omega_c^0} = 0.354(54) \text{ fm}^2$$

$$\langle r^2 \rangle_{M,\Omega_{cc}^+} = 0.148(21) \text{ fm}^2$$

$$\mu_{\Sigma_c^0} = -0.852(133) \mu_N$$

$$\mu_{\Sigma_c^{0+}} = 1.569(253) \mu_N$$

$$\mu_{\Xi_{cc}^+} = 0.411(15) \mu_N$$

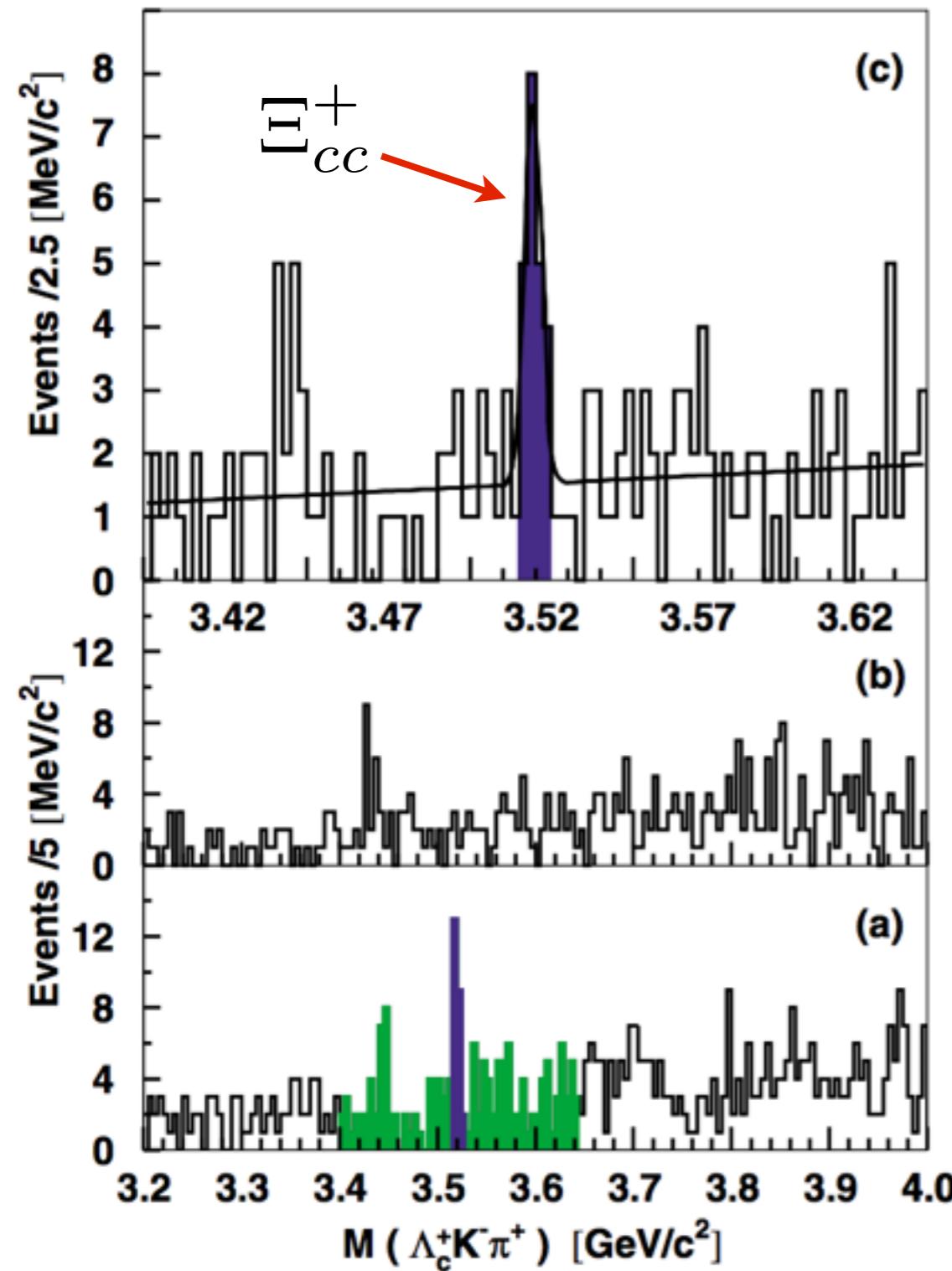
$$\mu_{\Omega_c^0} = -0.608(45) \mu_N$$

$$\mu_{\Omega_{cc}^+} = 0.405(13) \mu_N$$

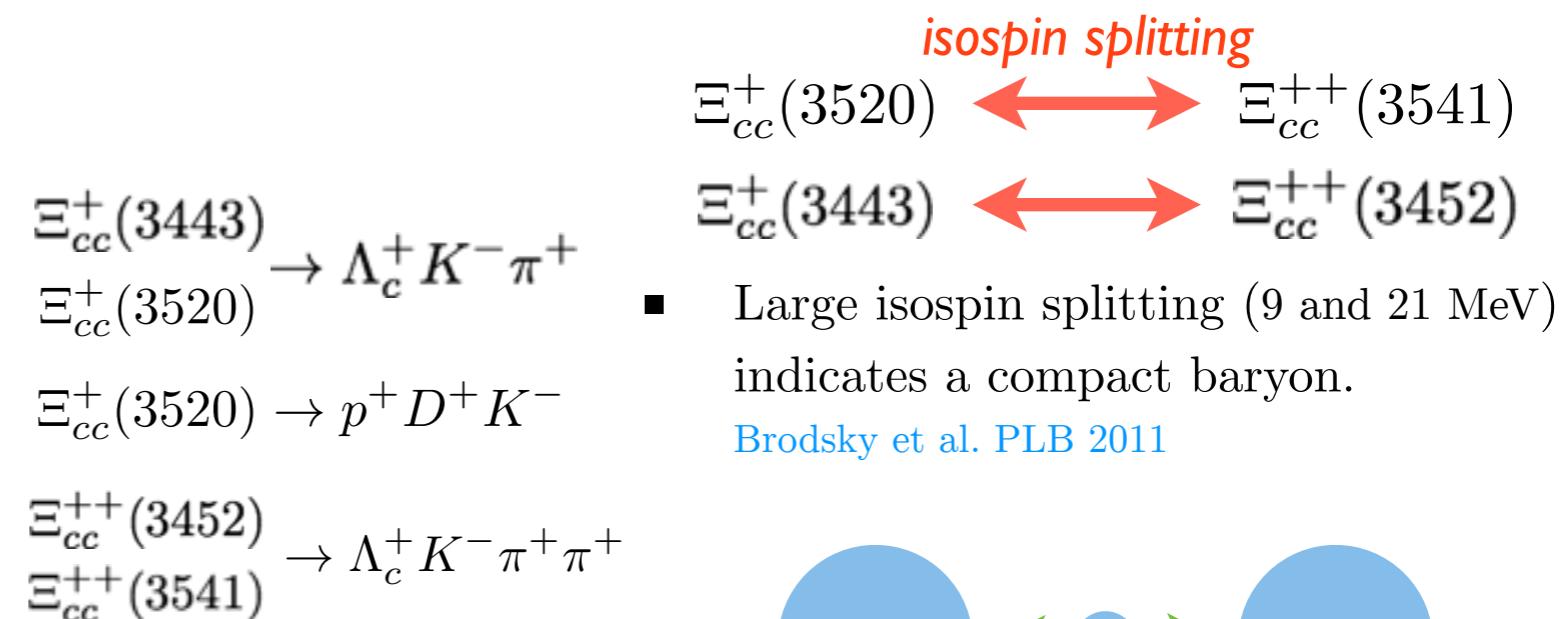
- Double-quark contribution dominates
- Heavy quarks shift centre of mass (CoM) closer to themselves

# DOUBLY CHARMED $\Xi_{CC}$

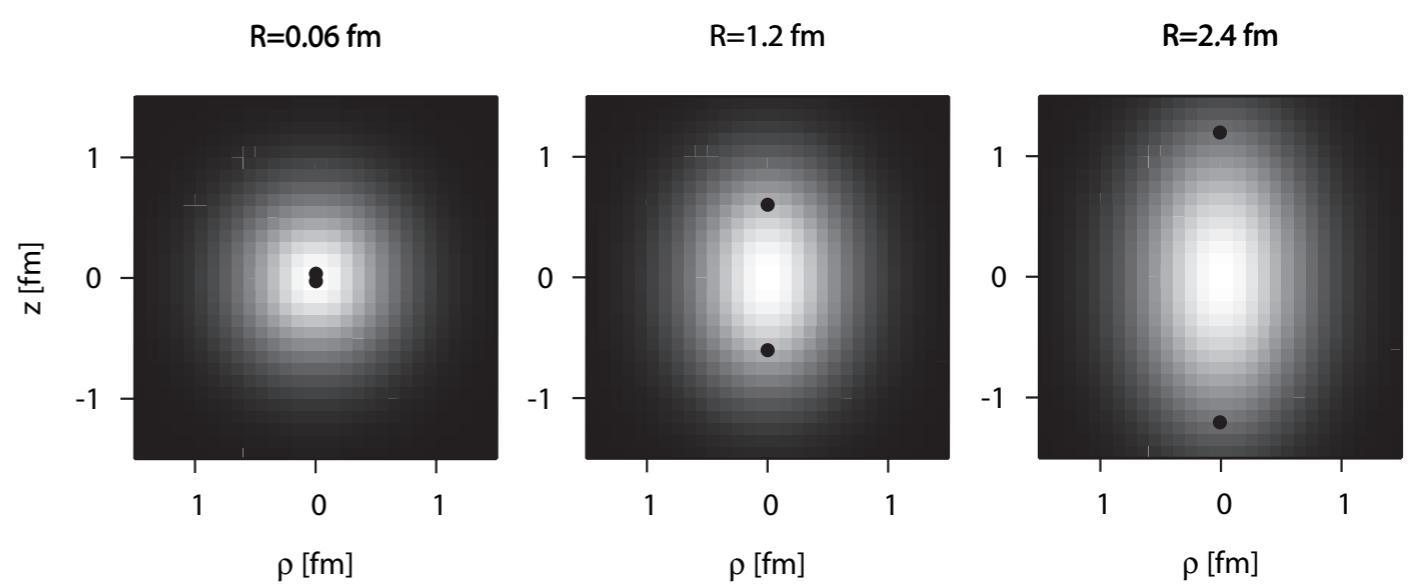
SELEX Collaboration (2002)



- $\Xi_{cc}^+$  omitted from PDG summary table.



- A. Yamamoto, H. Suganuma, H. Iida shows light quark is situated in the bright region  
 Phys. Rev. D 77, 014036 (2008)



# RESULTS

*comparison with other calculations  
(magnetic moments)*

	Our result		[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
	Lin. fit	Quad. fit									
$\mu_{\Sigma_c^0}$	-0.852(133)	-1.073(269)	-1.78	-1.04	-	-1.043	-1.60	-1.391	-1.17	-1.015	-1.6(2)
$\mu_{\Sigma_c^{++}}$	1.569(253)	2.220(505)	3.07	1.76	-	1.679	2.20	2.44	2.18	2.279	2.1(3)
$\mu_{\Xi_{cc}^+}$	0.411(15)	0.425(29)	0.94	0.72	$0.785^{+0.050}_{-0.030}$	0.722	0.84	0.774	0.77	-	-
$\mu_{\Omega_c^0}$	-0.608(45)	-0.639(88)	-0.90	-0.85	-	-0.774	-0.90	-0.85	-0.92	-0.960	-
$\mu_{\Omega_{cc}^+}$	0.405(13)	0.413(24)	0.74	0.67	$0.635^{+0.012}_{-0.015}$	0.668	0.697	0.639	0.70	0.785	-

- **Bottom line:** signs match but LQCD results underestimate the mag. moms (or other models overestimate)

[1] B. Julia Diaz et al., Rel. **QM**, hep-ph/0401096

[2] Faessler et al., Rel. **3QM**, hep-ph/0602193

[3] C. Albertus et al., **NRQM**, hep-ph/0610030

[4] Berthonas et al., **Bag Model**, hep-ph/1209.2900

[5] N. Sharma et al., **XQM**, hep-ph/1003.4338

[6] N. Barik et al., **indep-QM**, PRD 28 (1983)

[7] S. Kumar et al., **eff. mass and screened charge**, J.Phys G31 (2005)

[8] B. Patel et al., **hyper central model**, hep-ph/0710.3828

[9] S.-L. Zhu et al., **QCD spectral SR**, hep-ph/9708411

# *CONCLUSIONS & OUTLOOK*

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- Conclusions

- Charm quark shrinks the baryon's size, they are compact!
  - Observables are systematically small compared to the that of i.e. proton's.
- CoM is closer to Charm quark(s).
  - $E_{cc}$  is peculiar
  - Doubly represented quarks have the dominant contribution.

- Outlook

- Physical point calculation on  $\kappa_{ud} = 0.13781$  PAC-CS lattices
- Invest more on excited state analysis

*THANK YOU*

# *BACKUP SLIDES*

# *1S STATIC MASSES*

$\kappa_{val}^{u,d}$	$m_{\eta_c}$	$m_{J/\Psi}$	$m_D$	$m_{D^*}$	$m_{D_s}$	$m_{D_s^*}$
	[GeV]	[GeV]	[GeV]	[GeV]	[GeV]	[GeV]
Lin. Fit	2.979(2)	3.063(3)	1.895(6)	2.021(13)	2.018(4)	2.138(7)
Exp.	2.980	3.097	1.865	2.007	1.968	2.112
PACS-CS [17]	2.986(1)(13)	3.094(1)(14)	1.871(10)(8)	1.994(11)(9)	1.958(2)(9)	2.095(3)(10)

	$1S \, M_{\eta_c, J/\psi}$	$1S \, M_{D, D^*}$	$1S \, M_{D_s, D_s^*}$
This work	3.042(3)	1.990(42)	2.108(6)
Exp.	3.068	1.963	2.076
PACS-CS	3.067(15)	1.972(11)	2.061(12)

# *BARYON MASSES*

$\kappa_{val}^{u,d}$	$m_{\Sigma_c}$	$m_{\Omega_c}$	$m_{\Xi_{cc}}$	$m_{\Omega_{cc}}$
Lin. Fit	2.553(18)	2.740(24)	3.660(14)	3.755(18)
Exp.	2.455	2.695	3.519	-
PACS-CS [18]	2.467(39)(11)	2.673(5)(12)	3.603(15)(16)	3.704(5)(16)